Oscillations

Question1

A particle executing simple harmonic motion with amplitude A has the same potential and kinetic energies at the displacement

[NEET 2024 Re]

Options:

A.

2√A

B.

A/2

C.

A/√2

D.

A√2

Answer: C

Solution:

Potential energy $= \frac{1}{2}kx^2$

Kinetic energy $= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to given condition

$$\frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$k2x^2 = kA^2$$

$$x^2 = \frac{A^2}{2}$$

$$x = \frac{A}{\sqrt{2}}$$

Question2

The two-dimensional motion of a particle, described by $\vec{r} = (\hat{i} + 2\hat{j})A$ cos ω t is a/an:

A. parabolic path
B. elliptical path
C. periodic motion
D. simple harmonic motion
Choose the correct answer from the options given below:

[NEET 2024 Re]

Options:

A.

B, C and D only

B.

A, B and C only

C.

A, C and D only

D.

C and D only

Answer: D

Solution:

$\vec{r} = (\hat{i} + 2\hat{j}) A \cos \omega t$

$x = A \cos \omega t$

$y = 2A \cos \omega t$

$y = 2x$

The path is straight line.

The motion is SHM and periodic as

$\frac{dr}{dt} = -(\hat{i} + 2\hat{j}) \omega A \sin \omega t$

$\frac{d^2r}{dt^2} = -(\hat{i} + 2\hat{j}) \omega^2 A \cos \omega t$

$\vec{a} = -\omega^2 \vec{r}$

Question3

If x = 5sin (πt + π/3) m represents the motion of a particle executing simple harmonic motion, the amplitude and time period of motion, respectively, are

[NEET 2024]

Options:

A.

5cm, 2 s

B.

5m, 2 s

C.

5cm, 1 s

D.

5m, 1 s

Answer: B

Solution:

$$x = 5sin\left(\pi t + \frac{\pi}{3}\right)\text{m}$$

Amplitude $= 5\text{m}$

$$\omega = \pi = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\pi} = 2\ s$$

Question4

If the mass of the bob in a simple pendulum is increased to thrice its original mass and its length is made half its original length, then the new time period of oscillation is x/2 times its original time period. Then the value of x is:

[NEET 2024]

Options:

A.

√3

B.

√2

C.

2√3

D.

4

Answer: B

Solution:

$$T' = 2\pi\sqrt{\frac{\ell'}{g}} \text{ where } \ell' = \frac{\ell}{2}$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

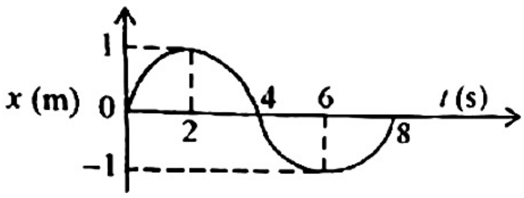
$$T' = \frac{x}{2}T$$

$$2\pi\sqrt{\frac{\ell}{2g}} = \frac{x}{2}2\pi\sqrt{\frac{\ell}{g}}$$

$$\frac{1}{\sqrt{2}} = \frac{x}{2} \Rightarrow x = \sqrt{2}$$

Question5

The x - t graph of a particle performing simple harmonic motion is shown in the figure. The acceleration of the particle at t = 2 s is



[NEET 2023]

Options:

A.

$-\frac{\pi^2}{8} \text{m } s^{-2}$

B.

$\frac{\pi^2}{16} \text{m } s^{-2}$

C.

$-\frac{\pi^2}{16} \text{ms}^{-2}$

D.

$\frac{\pi^2}{8} \text{m } s^{-2}$

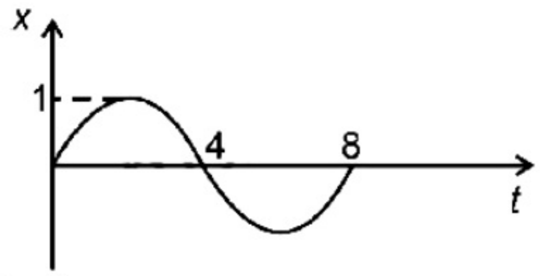
Answer: C

Solution:

Solution:

Position of particle as function of time

$x = A\sin\omega t$



From figure,

$A = 1$

$\omega = \frac{2\pi}{8}$

$x = \sin\frac{\pi}{4}t$

$v = \frac{dx}{dt}$

$v = \frac{\pi}{4}\cos\frac{\pi}{4}t$

$a = \frac{dv}{dt}$

$a = -\frac{\pi^2}{16}\sin\frac{\pi}{4}t$

at $t = 2\ s$

$a = -\frac{\pi^2}{16}\mathrm{m}/s^2$

Question6

A simple pendulum oscillating in air has a period of √3 s. If it is completely immersed in non-viscous liquid, having density (1/4)th of the material of the bob, the new period will be :-

[NEET 2023 mpr]

Options:

A.

$2\sqrt{3}$ s

B.

$\frac{2}{\sqrt{3}}s$

C.

2 s

D.

$\frac{\sqrt{3}}{2} s$

Answer: C

Solution:

Solution:

$$T_{air} = 2\pi\sqrt{\frac{\ell}{g}} = \sqrt{3}\ sec$$

$$\text{In Liquid} \rightarrow g_{net} = g\left(1 - \frac{\rho}{\sigma}\right)$$

$\rho =$ density of liquid

$\sigma =$ density of material of bob

$$\text{so } T_{Lqq} = 2\pi\sqrt{\left(\frac{\ell}{g_{net}}\right)} = 2\pi\sqrt{\frac{\ell}{g\left(1 - \frac{\rho}{\sigma}\right)}}$$

$$T_{Liq} = \frac{T_{air}}{\sqrt{1 - \frac{\rho}{\sigma}}} = \frac{\sqrt{3}}{\sqrt{1 - \frac{1}{4}}} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}} = 2\ sec$$

Question7

Two pendulums of length 121cm and 100cm start vibrating in phase. At some instant, the two are at their mean position in the same phase. The minimum number of vibrations of the shorter pendulum after which the two are again in phase at the mean position is:
[NEET-2022]

Options:

A. 11

B. 9

C. 10

D. 8

Answer: A

Solution:

$T = 2\pi \sqrt{\frac{L}{g}}$

Let n_1 and n_2 be integer.

$n_1 T_1 = n_2 T_2$

$2\pi n_1 \sqrt{\frac{1.21}{g}} = 2\pi n_2 \sqrt{\frac{1.00}{g}}$

$\Rightarrow \frac{n_2}{n_1} = \frac{11}{10}$

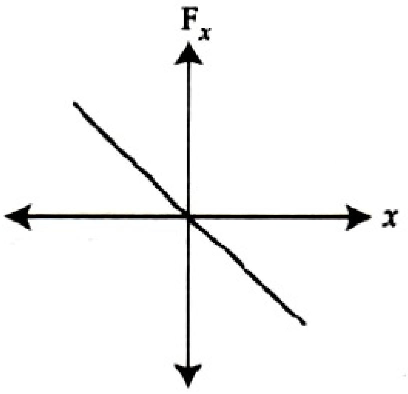
∴ After completion of 11th oscillation of shorter pendulum, it will be in phase with longer pendulum.

Question8

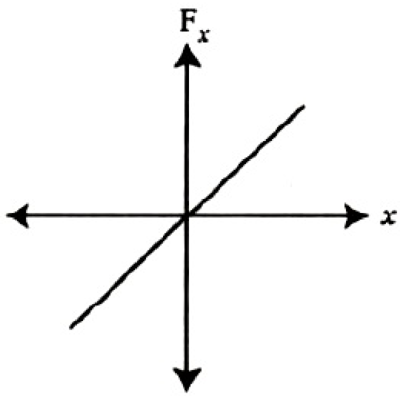
The restoring force of a spring with a block attached to the free end of the spring is represented by
[NEET Re-2022]

Options:

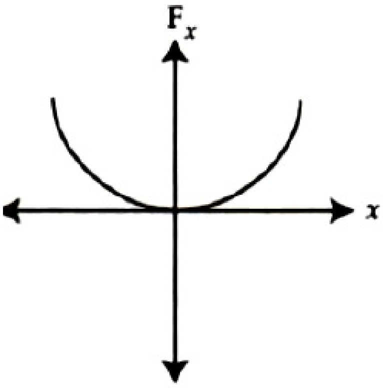
A.



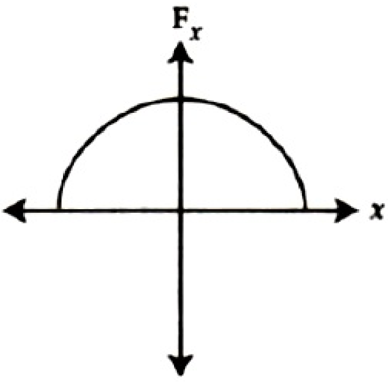
B.



C.



D.

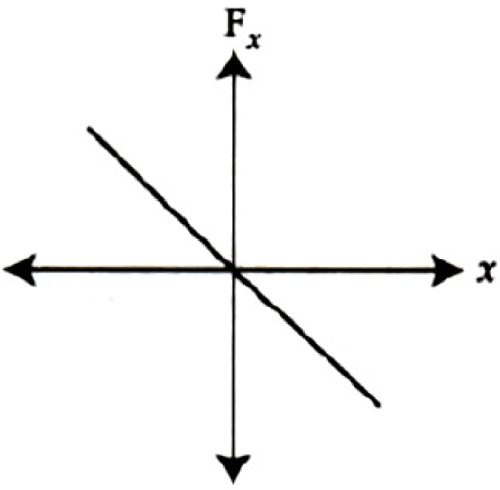


Answer: A

Solution:

Solution:

$F = -kx$



Question9

Identify the function which represents a non periodic motion. [NEET Re-2022]

Options:

A. $\sin(\omega t + \pi / 4)$

B. $e^{-\omega t}$

C. $\sin \omega t$

D. $\sin \omega t+\cos \omega t$

Answer: B

Solution:

Solution:

Periodic motion is represented by sin& cosine (harmonic functions) functions.

$e^{-\omega t}$ is not harmonic function

Question10

A body is executing simple harmonic motion with frequency 'n', the frequency of its potential energy is [NEET 2021]

Options:

A. n

B. 2n

C. 3n

D. 4n

Answer: B

Solution:

Solution:
Equation of displacement of particle executing SHM is given by $x = A\sin(\omega t+\phi)$.......(I)
Potential energy of particle executing SHM is given by

$U = \frac{1}{2}kx^2$

$= \frac{1}{2}kA^2\sin^2(\omega t+\phi)$ ....(II)

From I and II, it is clear that
Time period of $x = A\sin(\omega t+\phi)$ is

$T_1 = \frac{2\pi}{\omega} \Rightarrow$ frequency $n_1 = \frac{\omega}{2\pi}$

while time period of $x^2 = A^2\sin^2(\omega t+\phi)$ is

$T_2 = \frac{\pi}{\omega} \Rightarrow$ frequency $n_2 = \frac{\omega}{\pi}$

Hence $n_2 = 2n_1$

Question11

The phase difference between displacement and acceleration of a particle in a simple harmonic motion is:

(2020)

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Options:

A. $\frac{3\pi}{2}$rad

B. $\frac{\pi}{2}$rad

C. zero

D. π rad

Answer: D

Solution:

Solution:
(d) Displacement equation of a SHM
$y = A\sin(\omega t + \varphi)$
$\therefore$ Velocity, $v = \frac{dy}{dt} = A\omega\cos(\omega t + \varphi)$
Acceleration, $a = \frac{dv}{dt}$
or, $a = -A\omega^2\sin(\omega t + \varphi)$
$\therefore a = A\omega^2\sin(\omega t + \varphi + \pi)$
Hence, phase difference between displacement and acceleration is π.

Question12

The displacement of a particle executing simple harmonic motion is given by $y = A_0 + A\sin\omega t + B\cos\omega t$. Then the amplitude of its oscillation is given by
(NEET 2019)

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Options:

A. $A + B$

B. $A_0 + \sqrt{A^2 + B^2}$

C. $\sqrt{A^2 + B^2}$

D. $\sqrt{A_0^2 + (A + B)^2}$

Answer: C

Solution:

Solution:
$y = A_0 + A\sin\omega t + B\cos\omega t$

or $(y - A_0) = A \sin \omega t + B \cos \omega t$
or $y' = A \sin \omega t + B \cos \omega t$
$= A \cos\left(\frac{\pi}{2} - \omega t\right) + B \cos \omega t$
Amplitude $= \sqrt{A^2 + B^2 + 2AB \cos \frac{\pi}{2}}$ $\left[\because \phi = \frac{\pi}{2}\right]$
$= \sqrt{A^2 + B^2}$

Question13

Average velocity of a particle executing SHM in one complete vibration is
(NEET 2019)

Options:

A. zero

B. $\frac{A\omega}{2}$

C. $A\omega$

D. $\frac{A\omega^2}{2}$

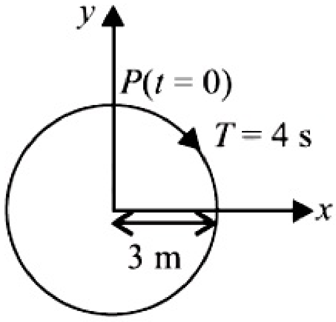
Answer: A

Solution:

Solution:
since the displacement for a complete vibration is zero, therefore the average velocity will be zero.

Question14

The radius of circle, the period of revolution, initial position and sense of revolution are indicated in the figure.



y -projection of the radius vector of rotating particle P is
(NEET 2019)

Options:

A. $y(t) = 3\cos\left(\frac{\pi t}{2}\right)$, where y in m

B. $y(t) = -3\cos 2\pi t$, where y in m

C. $y(t) = 4\sin\left(\frac{\pi t}{2}\right)$, where y in m

D. $y(t) = 3\cos\left(\frac{3\pi t}{2}\right)$, where y in m

Answer: A

Solution:

Solution:
Here $T = 4s, A = 3m$
Time period $T = \frac{2\pi}{\omega} \Rightarrow 4 = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{\pi}{2}$
When the time is noted from the extreme position,
So, $y = A\cos(\omega t) \Rightarrow y = 3\cos\left(\frac{\pi}{2}t\right)$

Question15

The distance covered by a particle undergoing SHM in one time period is (amplitude $= A$)
(OD NEET 2019)

Options:

A. zero

B. A

C. 2A

D. 4A

Answer: D

Solution:

Let at $t = 0$
particle is at point P and going towards point Q for one time period $= PQ + QP + PR + RP$
$= A + A + A + A$
$= 4A$

Question16

A pendulum is hung from the roof of a sufficiently high building and is

moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20ms^{-2} at a distance of 5m from the mean position. The time period of oscillation is
(NEET 2018)

Options:

A. 2πs

B. π s

C. 2s

D. 1 s

Answer: B

Solution:

Solution:
Magnitude of acceleration of a particle moving in a SHM is, $|a| = \omega^2 y$; where y is amplitude.
$\Rightarrow 20 = \omega^2(5) \Rightarrow \omega = 2\,\text{rad}\,s^{-1}$
$\therefore$ Time period of oscillation, $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi s$

Question17

A spring of force constant k is cut into lengths of ratio 1 : 2 : 3. They are connected in series and the new force constant is k′.Then they are connected in parallel and force constant is.Then k′ : k′′ is
(2017 NEET)

Options:

A. 1 : 9

B. 1 : 11

C. 1 : 14

D. 1 : 6

Answer: B

Solution:

Solution:
Let us assume, the length of spring be l .
When we cut the spring into ratio of length 1 : 2 : 3,
we cut three springs of lengths $\frac{l}{6}, \frac{2l}{6}$ and $\frac{3l}{6}$ with force constant

$\therefore k_1 = \frac{kl}{l_1} = \frac{kl}{l / 6} = 6k,\ k_2 = \frac{kl}{l_2} = \frac{kl}{2l / 6} = 3k$

$k_3 = \frac{kl}{l_3} = \frac{kl}{3l / 6} = 2k$

When connected in series,

$\frac{1}{k'} = \frac{1}{6k} + \frac{1}{3k} + \frac{1}{2k} = \frac{1 + 2 + 3}{6k} = \frac{1}{k}$

$\therefore k' = k$

When connected in parallel,$k'' = 6k + 3k + 2k = 11k$

$\frac{k'}{k''} = \frac{k}{11k} = \frac{1}{11}$

Question18

A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is
(2017 NEET)

Options:

A. $\frac{\sqrt{5}}{2\pi}$

B. $\frac{4\pi}{\sqrt{5}}$

C. $\frac{2\pi}{\sqrt{3}}$

D. $\frac{\sqrt{5}}{\pi}$

Answer: B

Solution:

Solution:
Given, A = 3 cm, x = 2 cm
The velocity of a particle in simple harmonic motion is given as
$v = \omega\sqrt{A^2 - x^2}$
and magnitude of its acceleration is
$a = \omega^2 x$
Given, $|v| = |a|$
$\therefore \omega\sqrt{A^2 - x^2} = \omega^2 x$
$\omega x = \sqrt{A^2 - x^2}$ or $\omega^2 x^2 = A^2 - x^2$
$\omega^2 = \frac{A^2 - x^2}{x^2} = \frac{9 - 4}{4} = \frac{5}{4}$
$\omega = \frac{\sqrt{5}}{2}$
Time period,$T = \frac{2\pi}{\omega} = 2\pi \cdot \frac{2}{\sqrt{5}} = \frac{4\pi}{\sqrt{5}} s$

Question19

A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. The value of m in kg is
(2016 NEET Phase-II)

Options:

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. $\frac{16}{9}$

D. $\frac{9}{16}$

Answer: D

Solution:

Solution:
Time period of spring - block system,
$T = 2\pi\sqrt{\frac{m}{k}}$
For given spring, $T \propto \sqrt{m}$
$\frac{T_1}{T_2} = \sqrt{\frac{m_1}{m_2}}$
Here, $T_1 = 3s, m_1 = m, T_2 = 5s, m_2 = m + 1, m = ?$
$\frac{3}{5}\sqrt{\frac{m}{m+1}}$ or $\frac{9}{25} = \frac{m}{m+1}$
$25m = 9m + 9 \Rightarrow 16m = 9 \Rightarrow m = \frac{9}{16}kg$

Question20

A particle is executing SHM along a straight line. Its velocities at distances x_1 and x_2 from the mean position are V_1 and V_2, respectively. Its time period is
(2015)

Options:

A. $2\pi\sqrt{\frac{V_1^2 + V_2^2}{x_1^2 + x_2^2}}$

B. $2\pi\sqrt{\frac{V_1^2 - V_2^2}{x_1^2 - x_2^2}}$

C. $2\pi\sqrt{\frac{x_1^2 + x_2^2}{V_1^2 + V_2^2}}$

D. $2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$

Answer: D

Solution:

Solution:

In SHM, velocities of a particle at distances x_1 and x_2 from mean position are given by

$V_1^2 = \omega^2(a^2 - x_1^2)$........(i)

$V_2^2 = \omega^2(a^2 - x_2^2)$....(ii)

From equations (i) and (ii), we get

$V_1^2 = V_2^2 = \omega^2(x_2^2 - x_1^2)$

$\omega = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}}$

$\therefore T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$

Question21

A particle is executing a simple harmonic motion. Its maximum acceleration is a and maximum velocity is p. Then, its time period of vibration will be (2015)

Options:

A. $\frac{\beta^2}{\alpha}$

B. $\frac{2\pi\beta}{\alpha}$

C. $\frac{\beta^2}{\alpha^2}$

D. $\frac{\alpha}{\beta}$

Answer: B

Solution:

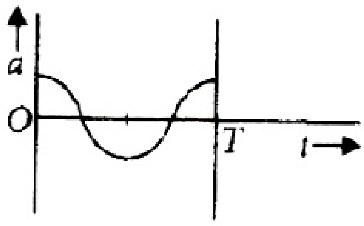
If A and ω be amplitude and angular frequency of vibration, then
$\alpha = \omega^2 A$....(i)
and $\beta = \omega A$.....(ii)
Dividing eqn. (i) by eqn. (ii), we get
$\frac{\alpha}{\beta} = \frac{\omega^2 A}{\omega A} = \omega$
∴Time period of vibration
$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\alpha}{\beta}} = \frac{2\pi\beta}{\alpha}$

Question22

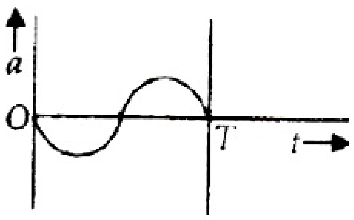
The oscillation of a body on a smooth horizontal surface is represented by the equation,
$X = A\cos(\omega t)$
Where X = displacement at time t
ω = frequency of oscillation
Which one of the following graphs shows correctly the variation a with t ?
Here a = acceleration at time
T = time period
(2014)

Options:

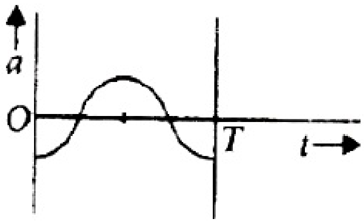
A.



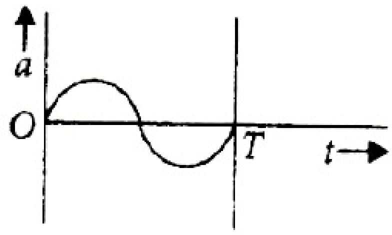
B.



C.



D.



Answer: C

Solution:

Solution:
Here, $X = A\cos\omega t$
$\therefore$ Velocity, $v = \frac{dX}{dt} = \frac{d}{dt}(A\cos\omega t) = -A\omega\sin\omega t$
Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}(-A\omega\sin\omega t) = -A\omega^2\cos\omega t$
Hence the variation of a with t is correctly shown by graph (c).

Question23

A particle of mass m oscillates along x -axis according to equation $x = a\sin\omega t$. The nature of the graph between momentum and displacement of the particle is (KN NEET 2013)

Options:

A. Circle

B. Hyperbola

C. Ellipse

D. Straight line passing through origin.

Answer: C

Solution:

$x = a\sin\omega t$ or $\frac{x}{a} = \sin\omega t$ ...(i)
Velocity, $v = \frac{dx}{dt} = a\omega\cos\omega t$
$\frac{v}{a\omega} = \cos\omega t$ ....(ii)
Squaring and adding (i) and (ii), we get
$\frac{x^2}{a^2} + \frac{v^2}{a^2\omega^2} = \sin^2\omega t + \cos^2\omega t$
$\frac{x^2}{a^2} + \frac{v^2}{a^2\omega^2} = 1$
It is an equation of ellipse.
Hence, the graph between velocity and displacement is an ellipse.
Momentum of the particle $= mv$

∴ The nature of graph of the momentum and displacement is same as that of velocity and displacement.

Question24

Out of the following functions representing motion of a particle which represents SHM
(1) $y = \sin\omega t - \cos\omega t$
(2) $y = \sin^3\omega t$
(3) $y = 5\cos\left(\frac{3\pi}{2} - 3\omega t\right)$
(4) $y = 1 + \omega t + \omega^2 t^2$
(2011)

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Options:

A. Only (1)

B. Only (4) does not represent SHM

C. Only (1) and (3)

D. Only (1) and (2)

Answer: C

Solution:

Solution:
$y = \sin\omega t - \cos\omega t$
$\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right] = \sqrt{2}\sin\left(\omega t - \frac{\pi}{2}\right)$
It represents a SHM with time period, $T = \frac{2\pi}{\omega}$.
$y = \sin^2\omega t = \frac{1}{4}[3\sin\omega t - \sin 3\omega t]$
It represents a periodic motion with time period $T = \frac{2\pi}{\omega}$ but not SHM.

$y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$
$= 5\cos\left(3\omega t - \frac{3\pi}{4}\right) \quad [\because \cos(-\theta) = \cos\theta]$
It represents a SHM with time period, $T = \frac{2\pi}{3\omega}$.
$y = 1 + \omega t + \omega^2 t^2$
It represents a non-periodic motion. Also it is not physically acceptable as the $y \to \infty$ as $t \to \infty$

Question25

Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite directions when their displacement is half of the

amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. The phase difference is (2011 Mains)

Options:

A. $\frac{\pi}{6}$

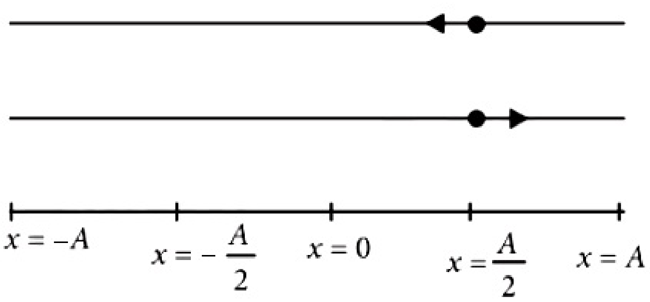
B. 0

C. $\frac{2\pi}{3}$

D. π

Answer: C

Solution:



The time taken by the particle to travel from

$x = 0$ to $x = \frac{A}{2}$ is $\frac{T}{12}$

The time taken by the particle to travel from

$x = A$ to $x = \frac{A}{2}$ is $\frac{T}{6}$

Time difference $= \frac{T}{6} + \frac{T}{6} = \frac{T}{3}$

Phase difference, $\phi = \frac{2\pi}{T} \times$ Time difference

$= \frac{2\pi}{T} \times \frac{T}{3} = \frac{2\pi}{3}$

Question26

The displacement of a particle along the x axis is given by $x = a\sin^2\omega t$. The motion of the particle corresponds to (2010)

Options:

A. simple harmonic motion of frequency $\frac{\omega}{\pi}$

B. simple harmonic motion of frequency $\frac{3\omega}{\pi}$

C. non simple harmonic motion

D. simple harmonic motion of frequency $\frac{\omega}{2\pi}$

Answer: C

Solution:

Solution:
$x = a\sin^2\omega t$

$= a\left(\frac{1-\cos2\omega t}{2}\right) \quad (\because \cos2\theta = 1 - 2\sin^2\theta)$

$= \frac{a}{2} - \frac{a\cos2\omega t}{2}$

$\therefore$ Velocity, $v = \frac{dx}{dt} = \frac{2\omega a\sin2\omega t}{2} = \omega a\sin2\omega t$

Acceleration, $a = \frac{dv}{dt} = 2\omega^2 a\cos2\omega t$

For the given displacement $xasin^2 = \omega t$, $a \propto -x$
is not satisfied. Hence, the motion of the particle is non simple harmonic motion.

Note : The given motion is a periodic motion with a time period $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Question27

The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be
(2010)

Options:

A. T

B. $\frac{T}{\sqrt{2}}$

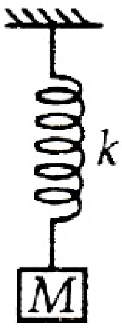
C. 2T

D. $\sqrt{2}T$

Answer: D

Solution:

Solution:
A mass M is suspended from a massless spring of spring constant k as shown in figure (a).

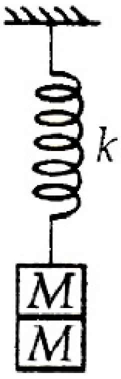


(a)

Then,

Time period of oscillation is $T = 2\pi\sqrt{\frac{M}{k}}$....(i)

When a another mass M is also suspended with it (a) as shown in figure (b).



(b)

Then, Time period of oscillation is

$T' = 2\pi\sqrt{\frac{M+M}{k}} = 2\pi\sqrt{\frac{2M}{k}}$

$= \sqrt{2}\left(2\pi\sqrt{\frac{M}{k}}\right) = \sqrt{2}T$ (Using (i))

Question28

A simple pendulum performs simple harmonic motion about x = 0 with an amplitude a and time period T. The speed of the pendulum at $x = \frac{a}{2}$ will be
(2009)

©

Options:

A. $\frac{\pi a}{T}$

B. $\frac{3\pi^2 a}{T}$

C. $\frac{\pi a\sqrt{3}}{T}$

D. $\frac{\pi a\sqrt{3}}{2T}$

Answer: C

Solution:

For simple harmonic motion, $v = \omega\sqrt{a^2 - x^2}$

When $x = \frac{a}{2}$, $v = \omega\sqrt{a^2 - \frac{a^2}{4}} = \omega\sqrt{\frac{3}{4}a^2}$

As $\omega = \frac{2\pi}{T}$, $\therefore v = \frac{2\pi}{T}\cdot\frac{\sqrt{3}}{2}a \Rightarrow v = \frac{\pi\sqrt{3}a}{T}$

Question29

Which one of the following equations of motion represents simple harmonic motion?
Where k, k_0, k_1 and a are all positive
(2009)

Options:

A. Acceleration $= -k(x + a)$

B. Acceleration $= k(x + a)$

C. Acceleration $= kx$

D. Acceleration $= -k_0x + k_1x^2$

Answer: A

Question30

Two simple harmonic motions of angular frequency 100 and 1000 rads^{-1} have the same displacement amplitude. The ratio of their maximum acceleration is
(2008)

Options:

A. $1 : 10^3$

B. $1 : 10^4$

C. $1 : 10$

D. $1 : 10^2$

Answer: D

Solution:

Here, $\omega_1 = 100 \text{ rad s}^{-1}$; $\omega_2 = 1000 \text{ rad s}^{-1}$

$a_{max_1} = \omega_1{}^2 A$

$a_{max_2} = \omega_2{}^2 A$

$$\therefore \frac{a_{max_1}}{a_{max_2}} = \frac{\omega_1{}^2}{\omega_2{}^2} = \frac{(100)^2}{(1000)^2} = \frac{1}{100}$$

Question31

A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T . The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is (2007)

Options:

A. $\frac{T}{8}$

B. $\frac{T}{12}$

C. $\frac{T}{2}$

D. $\frac{T}{4}$.

Answer: B

Solution:

$x(t) = a \sin \omega t$(from the equilibrium position)

At $x(t) = \frac{a}{2}$

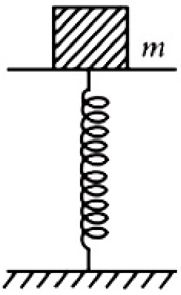
$a2 = a \sin(\omega t)$

$\Rightarrow \sin\left(\frac{\pi}{6}\right) = \sin(\omega t)$

or, $\frac{\pi}{6} = \frac{2\pi t}{T}\left[\because \omega = \frac{2\pi}{T}\right]$ or $t = \frac{T}{12}$

Question32

A mass of 2.0kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released, the mass executes a simple harmonic motion. The spring constant is 200N / m. What should be the minimum amplitude of the motion so that the mass gets detached from the pan (take $g = 10m / s^2$)



(2007)

Options:

A. 10.0cm

B. any value less than 12.0cm

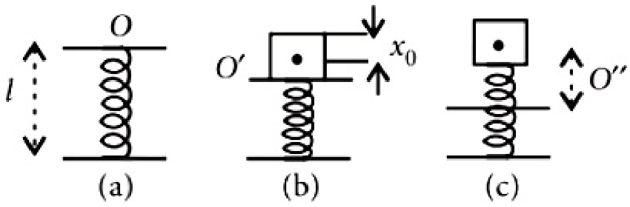
C. 4.0cm

D. 8.0cm.

Answer: A

Solution:

Solution:



The spring has a length l. When m is placed over it, the equilibrium position becomes O′. If it is pressed from O′ (the equilibrium position) to O", O′O" is the amplitude.

$\therefore OO' = \frac{mg}{k} = \frac{2 \times 10}{200} = 0.10m$

$mg = kx_0$

If the restoring force $mA\omega^2 > mg$, then the mass will move up with acceleration, detached from the pan.

i.e., $A > gk / m \Rightarrow A > \frac{20}{200} > 0.10m$

The amplitude >10cm.

i.e. the minimum is just greater than 10cm. (The actual compression will include x_0 also. But when talking of amplitude, it is always from the equilibrium position with respect to which the mass is oscillating.

Question33

The particle executing simple harmonic motion has a kinetic energy $K_0 \cos^2 \omega t$. The maximum values of the potential energy and the total energy are respectively
(2007)

Options:

A. $\frac{K_0}{2}$ and K_0

B. K_0 and $2K_0$

C. K_0 and K_0

D. 0 and $2K_0$

Answer: C

Solution:

Kinetic energy + potential energy = total energy
When kinetic energy is maximum, potential energy is zero and vice versa.
∴ Maximum potential energy = total energy.
$0 + K_0 = K_0 (K.E. + P.E. \text{ total energy})$

Question34

The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is (2007)

Options:

A. π

B. 0.707π

C. zero

D. 0.5π

Answer: D

Solution:

Solution:
Let $y = A \sin \omega t$
$\frac{dy}{dt} = A\omega \cos \omega t = A\omega \sin\left(\omega t + \frac{\pi}{2}\right)$
Acceleration $= -A\omega^2 \sin \omega t$
The phase difference between acceleration and velocity is $\frac{\pi}{2}$

Question35

A rectangular block of mass m and area of cross-section A floats in a liquid of density \ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T , then (2006)

Options:

A. $T \propto \frac{1}{\sqrt{m}}$

B. $T \propto \sqrt{\rho}$

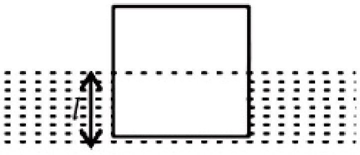
C. $T \propto \frac{1}{\sqrt{A}}$

D. $T \propto \frac{1}{\rho}$

Answer: C

Solution:

Solution:



Let l be the length of block immersed in liquid as shown in the figure. When the block is floating,
$\therefore mg = Al\rho m$
If the block is given vertical displacement y then the effective restoring force is
$F = -[A(l + y)\rho g - mg] = -[A(l + y)\rho g - Al\rho g]$
$= -Al\rho gy$
Restoring force $= -[Al\rho g]y$. As this F is directed towards equilibrium position of block, so if the block is left free, it will execute simple harmonic motion.
Here inertia factor = mass of block $= m$
Spring factor $= A\rho g$
$\therefore$ Time period, $T = 2\pi\sqrt{\frac{m}{A\rho g}}$ i.e., $T \propto \frac{1}{\sqrt{A}}$

Question36

The circular motion of a particle with constant speed is (2005)

Options:

A. periodic but not simple harmonic

B. simple harmonic but not periodic

C. period and simple harmonic

D. neither periodic nor simple harmonic.

Answer: A

Question37

A particle executing simple harmonic motion of amplitude 5cm has maximum speed of 31.4cm / s. The frequency of its oscillation is (2004)

Options:

A. 4Hz

B. 3Hz

C. 2Hz

D. 1Hz.

Answer: D

Solution:

$a = 5cm, v_{max} = 31.4cm / s$
$v_{max} = \omega a \Rightarrow 31.4 = 2\pi v \times 5$
$\Rightarrow 31.4 = 10 \times 3.14 \times v \Rightarrow v = 1Hz.$

Question38

Two springs of spring constant k_1 and k_2 are joined in series. The effective spring constant of the combination is given by (2004)

Options:

A. $\sqrt{k_1 k_2}$

B. $\frac{(k_1 + k_2)}{2}$

C. $k_1 + k_2$

D. $\frac{k_1 k_2}{(k_1 + k_2)}$

Answer: D

Solution:

Solution:
When the spring joined in series, the total extension in spring is

$\Rightarrow y = y_1 + y_2 = \frac{-F}{k_1} - \frac{F}{k_2}$

$\Rightarrow y = -F\left[\frac{1}{k_1} + \frac{1}{k_2}\right]$

Thus spring constant in this case becomes

$k = \frac{k_1 k_2}{k_1 + k_2}$

Question39

Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion? (2003)

Options:

A. When v is maximum, a is maximum.

B. Value of a is zero, whatever may be the value of v

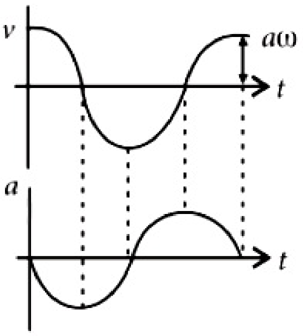
C. When v is zero, a is zero.

D. When v is maximum, a is zero.

Answer: D

Solution:

Solution:



In SHM $v = A\omega \sin\left(\omega t + \frac{\pi}{2}\right)$, $a = A\omega^2 \sin(\omega t + \pi)$. From this we can easily find out that when v is maximum, then a is zero.

Question40

The potential energy of a simple harmonic oscillator when the particle is half way to its end point is (2003)

Options:

A. $\frac{2}{3}E$

B. $\frac{1}{8}$E

C. $\frac{1}{4}$E

D. $\frac{1}{2}$E

Answer: C

Solution:

Solution:

Potential energy of simple harmonic oscillator $= \frac{1}{2} m\omega^2 y^2$

For $y = \frac{a}{2}$, P.E. $= \frac{1}{2} m\omega^2 \frac{a^2}{4}$

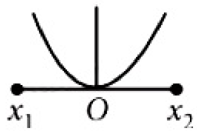
$\Rightarrow$ P.E. $= \frac{1}{4}\left(\frac{1}{2} m\omega^2 a^2\right) = \frac{E}{4}$

Question41

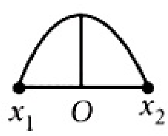
A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position beingO. Its potential energy is plotted. It will be as given below in the graph
(2003)

Options:

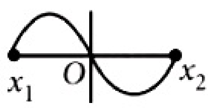
A.



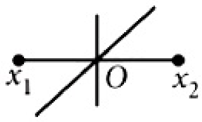
B.



C.



D.



Answer: A

Solution:

Solution:
Potential energy of particle performing SHM varies parabolically in such a way that at mean position it becomes zero and maximum at extreme position.

Question42

The time period of a mass suspended from a spring is T . If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be (2003)

Options:

A. $\frac{T}{4}$

B. T

C. $\frac{T}{2}$

D. 2T

Answer: C

Solution:

Solution:
Let k be the force constant of spring. If k' is the force constant of each part, then
$\frac{1}{k}=\frac{4}{k'} \Rightarrow k'=4k$
$\therefore$ Time period $=2\pi\sqrt{\frac{m}{4k}}=\frac{1}{2}\times 2\pi\sqrt{\frac{m}{k}}=\frac{T}{2}$

Question43

In case of a forced vibration, the resonance peak becomes very sharp when the (2003)

Options:

A. damping force is small

B. restoring force is small

C. applied periodic force is small

D. quality factor is small

Answer: A

Solution:

Solution:
Smaller damping gives a taller and narrower resonance peak.

Question44

Displacement between maximum potential energy position and maximum kinetic energy position for a particle executing simple harmonic motion is
(2002)

Options:

A. $\pm\frac{a}{2}$

B. +a

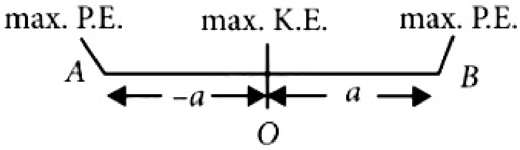
C. ±a

D. -1

Answer: C

Solution:

Solution:
For a simple harmonic motion between A and B, with O as the mean position, maximum kinetic energy of the particle executing SHM will be at O and maximum potential energy will be at A and B.



∴ Displacement between maximum potential energy and maximum kinetic energy is ±a.

Question45

When an oscillator completes 100 oscillations, its amplitude reduced to $\frac{1}{3}$ of initial value. What will be its amplitude, when it completes 200 oscillations?
(2002)

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Options:

A. $\frac{1}{8}$

B. $\frac{2}{3}$

C. $\frac{1}{6}$

D. $\frac{1}{9}$

Answer: D

Solution:

Solution:
This is a case of damped vibration as the amplitude of vibration is decreasing with time.
Amplitude of vibrations at any instant t is given by $a = a_0 e^{-bt}$, where a_0 is the initial amplitude of vibrations and b is the damping constant.

Now, when $t = 100T$, $a = \frac{a_0}{3}$[T is time period]

Let the amplitude be a′ at $t = 200T$
.i.e. after completing 200 oscillations.

$\therefore a = \frac{a_0}{3} = a_0 e^{-100T\,b}$ .....(i)

and $a' = a_0 e^{-200T\,b}$ .....(ii)

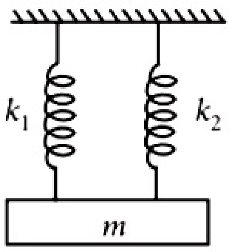
From (i), $\frac{1}{3} = e^{-100T\,b}$ $\therefore e^{-200T\,b} = \frac{1}{9}$

From(ii), $a' = a_0 \times \frac{1}{9} = \frac{a_0}{9}$

therefore The amplitude will be reduced to $\frac{1}{9}$ of initial value.

Question46

A mass is suspended separately by two different springs in successive order then time periods is t_1 and t_2 respectively. If it is connected by both spring as shown in figure then time period is t_0, the correct relation is



(2002)

Options:

A. $t_0^{\,2} = t_1^{\,2} + t_2^{\,2}$

B. $t_0^{\,-2} = t_1^{\,-2} + t_2^{\,-2}$

C. $t_0^{\,-1} = t_1^{\,-1} + t_2^{\,-1}$

D. $t_0 = t_1 + t_2$

Answer: B

Solution:

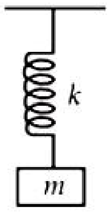


Figure 1

The time period of a spring mass system as shown in figure 1 is given by $T = 2\pi\sqrt{\frac{m}{k}}$, where k is the spring constant.

$\therefore t_1 = 2\pi\sqrt{\frac{m}{k_1}}$ .....(i)

and $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$ ....(ii)

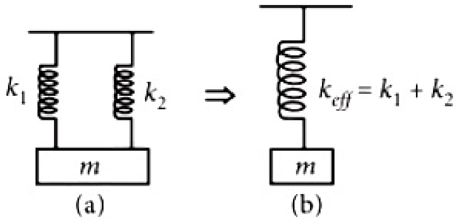


Figure 2

Now, when they are connected in parallel as shown in figure 2(a), the system can be replaced by a single spring of spring constant, $k_{eff} = k_1 + k_2$, as shown in figure 2(b)

[since $mg = k_1x + k_2x = k_{eff}x$]

$t_0 = 2\pi\sqrt{\frac{m}{k_{eff}}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$ ....(iii)

From(i), $\frac{1}{t_1^2} = \frac{1}{4\pi^2} \times \frac{k_1}{m}$ ......(iv)

From(ii), $\frac{1}{t_2^2} = \frac{1}{4\pi^2} \times \frac{k_2}{m}$ ......(v)

From(iii), $1t_0^2 = \frac{1}{4\pi^2} \times \frac{k_1 + k_2}{m}$ ......(vi)

(iv) +(v) $= \frac{1}{t_1^2} + \frac{1}{t_2^2} = \frac{1}{4\pi^2 m}(k_1 + k_2) = \frac{1}{t_0^2}$

$\therefore t_0^{-2} = t_1^{-2} + t_2^{-2}$

Question47

The total energy of particle performing SHM depends on (2001)

Options:

A. k, a, m

B. k, a

C. k, a, x

D. k, x.

Answer: B

Solution:

Solution:

Energy $= \frac{1}{2}m\omega^2a^2 = \frac{1}{2}ka^2$

Question48

Two masses M_A and M_B are hung from two strings of length l_A and l_B respectively. They are executing SHM with frequency relation $f_A = 2f_B$, then relation
(2000)

Options:

A. $l_A = \frac{l_B}{4}$, does not depend on mass

B. $l_A = 4l_B$, does not depend on mass

C. $l_A = 2l_B$ and $M_A = 2M_B$

D. $l_A = \frac{l_B}{2}$ and $M_A = \frac{M_B}{2}$

Answer: A

Solution:

$f_A = 2f_B$

$\Rightarrow \frac{1}{2\pi}\sqrt{\frac{g}{l_A}} = 2 \times \frac{1}{2\pi}\sqrt{\frac{g}{l_B}}$ or, $\frac{1}{l_A} = 4 \times \frac{1}{l_B}$

or, $l_A = \frac{l_B}{4}$, which does not depend on mass.

Question49

The bob of simple pendulum having length l, is displaced from mean position to an angular position θ with respect to vertical. If it is released, then velocity of bob at equilibrium position
(2000)

Options:

A. $\sqrt{2gl(1 - \cos\theta)}$

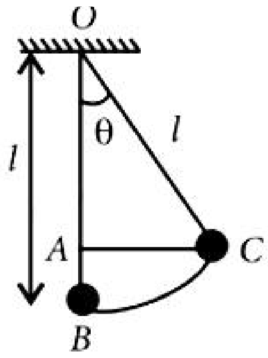
B. $\sqrt{2gl(1 + \cos\theta)}$

C. $\sqrt{2gl\cos\theta}$

D. $\sqrt{2gl}$

Answer: A

Solution:



In ΔOAC, $\cos\theta = \frac{A}{l}$
or, $OA = l\cos\theta$
$\therefore AB = l(1 - \cos\theta) = h$
At point, C, the velocity of bob $= 0$.
The vertical acceleration $= g$
$\therefore v^2 = 2gh$
or, $v = \sqrt{2gl(1 - \cos\theta)}$

Question50

Time period of a simple pendulum is 2 sec. If its length is increased by 4 times, then its time period becomes
(1999)

Options:

A. 8 sec

B. 12 sec

C. 16 sec

D. 4 sec

Answer: D

Solution:

Time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$

$\therefore \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$ or, $T_2 = 2T_1 = 4\,\text{sec}$

Question51

A particle, with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force F $\sin\omega t$. If the amplitude of the particle is maximum for $\omega = \omega_1$ and the energy of the particle is maximum for $\omega = \omega_2$, then
(1998, 1989)

Options:

A. $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$

B. $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$

C. $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_0$

D. $\omega_1 \neq \omega_0$ and $\omega_2 \neq \omega_0$

Answer: B

Solution:

Solution:
The amplitude and velocity resonance occurs at the same frequency.
At resonance, i.e., $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$, the amplitude and energy of the particle would be maximum.

Question52

Two simple pendulums of length 5m and 20m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed _______ oscillations.
(1998)

Options:

A. 2

B. 1

C. 5

D. 3

Answer: A

Solution:

Frequency of the pendulum

$\nu_{l=5} = \frac{1}{2\pi}\sqrt{\frac{g}{5}}; \nu_{l=20} = \frac{1}{2\pi}\sqrt{\frac{g}{20}}$

$\therefore \frac{\nu_{l=5}}{\nu_{l=20}} = \sqrt{\frac{20}{5}} = 2 \Rightarrow \nu_{l=5} = 2\nu_{l=20}$

As shorter length pendulum has frequency double the larger length pendulum. Therefore shorter pendulum should complete 2 oscillations before they will be again in phase.

Question53

A mass m is vertically suspended from a spring of negligible mass; the system oscillates with a frequency n. What will be the frequency of the system, if a mass 4m is suspended from the same spring?
(1998)

Options:

A. $\frac{n}{2}$

B. 4n

C. $\frac{n}{4}$

D. 2n

Answer: A

Solution:

Solution:

$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}; n' = \frac{1}{2\pi}\sqrt{\frac{k}{4m}}$

$\therefore n' = \frac{n}{2}$

Question54

If the length of a simple pendulum is increased by 2%, then the time period
(1997)

Options:

A. increases by 1%

B. decreases by 1%

C. increases by 2%

D. decreases by 2%.

Answer: A

Solution:

Solution:
$l_2 = 1.02 l_1$.

Time period $(T) = 2\pi \times \sqrt{\frac{l}{g}} \propto \sqrt{l}$

Therefore $\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{1.02 l_1}{l_1}} = 1.01$

Thus time period increased by 1%.

Question55

Two SHM's with same amplitude and time period, when acting together in perpendicular directions with a phase difference of $\frac{\pi}{2}$, give rise to (1997)

Options:

A. straight motion

B. elliptical motion

C. circular motion

D. none of these.

Answer: C

Solution:

$x = a \sin \omega t$

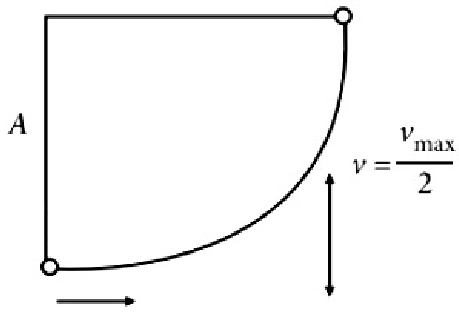
$y = a \sin\left(\omega t + \frac{\pi}{2}\right) = a \cos \omega t$

or, $\frac{x}{y} = \frac{\sin \omega t}{\cos \omega t} = \tan \omega t$ or, $\frac{x}{y} = \frac{x}{\sqrt{a^2 - x^2}}$,

or, $y^2 = a^2 - x^2$ or, $x^2 + y^2 = a^2$.
It is an equation of a circle.

Question56

A particle starts with S.H.M. from the mean position as shown in the figure. Its amplitude is A and its time period is T. At one time, its speed is half that of the maximum speed. What is this displacement?



(1996)

Options:

A. $\frac{2A}{\sqrt{3}}$

B. $\frac{3A}{\sqrt{2}}$

C. $\frac{\sqrt{2}A}{3}$

D. $\frac{\sqrt{3}A}{2}$

Answer: D

Solution:

Maximum velocity, $v_{max} = A\omega$

According to question, $\frac{v_{max}}{2} = \frac{A\omega}{2} = \omega\sqrt{A^2 - y^2}$

$\frac{A^2}{4} = A^2 - y^2 \Rightarrow y^2 = A^2 - \frac{A^2}{4} \Rightarrow y = \frac{\sqrt{3}A}{2}$

Question57

A linear harmonic oscillator of force constant 2×10^6 N / m and amplitude 0.01m has a total mechanical energy of 160 J. Its (1996)

Options:

A. P.E. is 160 J

B. P.E. is zero

C. P.E. is 100 J

D. P.E. is 120 J.

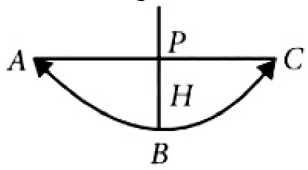
Answer: C

Solution:

Force constant $(k) = 2 \times 10^6 N / m$;
Amplitude $(x) = 0.01 m$ and total mechanical energy $= 160 J$.
Potential energy $= \frac{1}{2} kx^2 = \frac{1}{2} \times (2 \times 10^6)(0.01)^2$
$= 100 J$

Question58

A simple pendulum with a bob of mass m oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is g, then the velocity of the bob as it passes through B is



(1995)

Options:

A. mgH

B. $\sqrt{2gH}$

C. zero

D. 2gH .

Answer: B

Solution:

Solution:
Potential energy at A (or C) = Kinetic energy at B.
Thus, $\frac{1}{2} m v_B^2 = mgH$ or $v_B = \sqrt{2gH}$

Question59

In a simple harmonic motion, when the displacement is one-half the amplitude, what fraction of the total energy is kinetic?
(1995)

Options:

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. zero

D. $\frac{1}{4}$.

Answer: B

Solution:

Displacement $(x) = \frac{a}{2}$

Total energy $= \frac{1}{2}m\omega^2a^2$

and kinetic energy when displacement is (x)

$$= \frac{1}{2}m\omega^2(a^2 - x^2)$$

$$= \frac{1}{2}m\omega^2\left(a^2 - \left(\frac{a}{2}\right)^2\right) = \frac{3}{4}\left(\frac{1}{2}m\omega^2a^2\right)$$

Therefore fraction of the total energy at x,

$$= \frac{\frac{3}{4}\left(\frac{1}{2}m\omega^2a^2\right)}{\frac{1}{2}m\omega^2a^2} = \frac{3}{4}.$$

Question60

A body of mass 5kg hangs from a spring and oscillates with a time period of 2π seconds. If the ball is removed, the length of the spring will decrease by
(1994)

Options:

A. g / k metres

B. k / g metres

C. 2π metres

D. g metres.

Answer: D

Solution:

Solution:
Mass $(m) = 5kg$ and time period $(T) = 2\pi$ sec.

Therefore time period $T = 2\pi \times \sqrt{\frac{m}{k}} \Rightarrow \sqrt{\frac{5}{k}} = 1$

or $k = 5N / m$.

According to Hooke's Law, $F = -kl$.
Therefore decrease in length $(l) = -\frac{F}{k} = -\frac{5g}{5} = -g$ metres

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Question61

A particle executes S.H.M. along x -axis. The force acting on it is given by
(1994, 1988)

Options:

A. $A\cos(kx)$

B. Ae^{-kx}

C. Akx

D. $-Akx$.

Answer: D

Solution:

Solution:
For simple harmonic motion $\frac{d^2x}{dt^2} \propto -x$
Therefore force acting on the particle $= -Akx$.

Question62

A seconds pendulum is mounted in a rocket. Its period of oscillation will decrease when rocket is
(1994)

Options:

A. moving down with uniform acceleration

B. moving around the earth in geostationary orbit

C. moving up with uniform velocity

D. moving up with uniform acceleration.

Answer: D

Solution:

Period of oscillation $T = 2\pi\sqrt{\frac{l}{g}}$.
Therefore T will decrease when acceleration (g) increases. And g will increase when the rocket moves up with a uniform acceleration.

Question63

A loaded vertical spring executes S.H.M. with a time period of 4 sec. The difference between the kinetic energy and potential energy of this system varies with a period of
(1994)

Options:

A. 2 sec

B. 1 sec

C. 8 sec

D. 4 sec

Answer: A

Solution:

Solution:
Time period $= 4$ sec. In one simple harmonic oscillation, the same kinetic and potential energies are repeated two times. So the difference will be 2 seconds.

Question64

A body executes simple harmonic motion with an amplitude A . At what displacement from the mean position is the potential energy of the body is one fourth of its total energy?
(1993)

Options:

A. $\frac{A}{4}$

B. $\frac{A}{2}$

C. $\frac{3A}{4}$

D. Some other fraction of A

Answer: B

Solution:

$$P.E = \frac{1}{2}M\omega^2x^2 = \frac{1}{4}E = \frac{1}{4}\left(\frac{1}{2}M\omega^2A^2\right)$$

where total energy $E = \frac{1}{2}M\omega^2A^2 \quad \therefore x = \frac{A}{2}$

Question65

A simple harmonic oscillator has an amplitude A and time period T. The time required by it to travel from x = A to x = $\frac{A}{2}$ is
(1992)

Options:

A. $\frac{T}{6}$

B. $\frac{T}{4}$

C. $\frac{T}{3}$

D. $\frac{T}{2}$

Answer: A

Solution:

Solution:

For S.H.M., $x = A\sin\left(\frac{2\pi}{T}t\right)$

when $x = A$, $A = A\sin\left(\frac{2\pi}{T}t\right)$

$\therefore \sin\left(\frac{2\pi}{T}.t\right) = 1 \Rightarrow \sin\left(\frac{2\pi}{T}.t\right) = \sin\left(\frac{\pi}{2}\right)$

$\Rightarrow t = \left(\frac{T}{4}\right)$

When $x = \frac{A}{2}, \frac{A}{2} = A\sin\left(\frac{2\pi}{T}.t\right)$

or $\sin\frac{\pi}{6} = \sin\left(\frac{2\pi}{T}t\right)$ or $t = \left(\frac{T}{12}\right)$

Now, time taken to travel from $x = A$ to $x = \frac{A}{2}$

$= \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$

Question66

If a simple harmonic oscillator has got a displacement of 0.02m and acceleration equal to 2.0 ms^{-2} at any time, the angular frequency of the oscillator is equal to
(1992)

Options:

A. $10\,rad\,s^{-1}$

B. $0.1\,rad\,s^{-1}$

C. $100\,rad\,s^{-1}$

D. $1\,rad\,s^{-1}$

Answer: A

Solution:

Solution:
When a particle undergoes SHM, its acceleration is given by,
$\alpha = \omega^2 x$
Given, $\alpha = 2$, $x = 0.02$. Using these values
$\omega = \sqrt{\frac{a}{x}}$
or, $\omega = \sqrt{\frac{2}{0.02}}$
or, $\omega = 10rad\ / s$

Question67

A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration a, then the time period is given by $T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$ where g is equal to

(1991)

Options:

A. g

B. $g - a$

C. $g + a$

D. $\sqrt{(g^2 + a^2)}$

Answer: D

Solution:

The effective value of acceleration due to gravity is $\sqrt{(a^2 + g^2)}$.

Question68

A body is executing simple harmonic motion. When the displacements from the mean position is 4cm and 5cm, the corresponding velocities of the body is 10 cm / sec and 8 cm / sec. Then the timeperiod of the body is (1991)

Options:

A. 2π sec

B. $\frac{\pi}{2}$/sec

C. π sec

D. $\frac{3\pi}{2}$ sec

Answer: C

Solution:

Solution:

For simple harmonic motion velocity $v = \omega\sqrt{a^2 - x^2}$ at displacement x.

$10 = \omega\sqrt{a^2 - 16}$ .....(i)

$8 = \omega\sqrt{a^2 - 25}$ .....(ii)

$\frac{100}{\omega^2} = a^2 - 16$... (iii)

$\frac{64}{\omega^2} = a^2 - 25$ .....(iv)

$\therefore$ Equation (iii) − (iv) gives $\frac{36}{\omega^2} = 9$

$\Rightarrow \omega = 2$ rad / s or $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ sec

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Question69

The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement x from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is (1991)

Options:

A. $\frac{(a^2 - x^2\omega^2)}{x^2\omega^2}$

B. $\frac{x^2\omega^2}{(a^2 - x^2\omega^2)}$

C. $\frac{(a^2 - x^2)}{x^2}$

D. $\frac{x^2}{(a^2 - x^2)}$

Answer: C

Solution:

P.E., V $= \frac{1}{2}m\omega^2x^2$

and K . E ., T $= \frac{1}{2}m\omega^2(a^2 - x^2)$

$\therefore \frac{T}{V} = \frac{a^2 - x^2}{x^2}$

Question70

The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π results in the displacement of the particle along
(1990)

Options:

A. circle

B. figure of eight

C. straight line

D. ellipse

Answer: C

Solution:

$x = a\sin\omega t$
and $y = b\sin(\omega t + \pi) = -b\sin\omega t$
or $\frac{x}{a} = -\frac{y}{b}$ or $y = -\frac{b}{a}x$
It is an equation of straight line.

Question71

A mass m is suspended from the two coupled springs connected in

series. The force constant for springs are k_1 and k_2. The time period of the suspended mass will be
(1990)

Options:

A. $T = 2\pi\sqrt{\frac{m}{k_1 - k_2}}$

B. $T = 2\pi\sqrt{\frac{mk_1k_2}{k_1 + k_2}}$

C. $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

D. $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$

Answer: D

Solution:

The effective spring constant of two springs in series is $k = \frac{k_1k_2}{k_1 + k_2}$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$
